Math 55 Quiz 1 DIS 106

Name: _____ 31 Jan 2022

1. Let $M(x, y)$ be the statement "Person x knows how to play instrument y." Express each of these sentences in terms of $M(x, y)$, quantifiers, and logical connectives, where the domain for x consists of everyone in the world and for y consists of all instruments known to mankind.

   (a) Alvin knows how to play piano and guitar, but not the drums. [2 points]

   (b) If one knows how to play trumpet or trombone, then they know how to play tuba. [2 points]

   (c) Every instrument has someone who knows how to play it; but there exists an instrument which only one person in the world knows how to play it. [3 points]

   (a) $M(\text{Alvin, piano}) \wedge M(\text{Alvin, guitar}) \wedge \neg M(\text{Alvin,drums})$

   (b) $\forall x((M(x, \text{trumpet}) \vee M(x, \text{trombone})) \rightarrow M(x, \text{tuba}))$

   (c) $(\forall y \exists x M(x, y)) \wedge (\exists y \exists x (M(x, y) \wedge \forall x'((x' \neq x) \rightarrow \neg M(x', y))))$

2. Prove that if x is a rational number, then $2x + 1$ is a rational number. [3 points]

Suppose x is a rational number, then there exists integers m and n such that $x = \frac{m}{n}$. Then $2x + 1 = 2\frac{m}{n} + 1 = \frac{2m+n}{n}$, where $2m + n$ and n are integers. Hence $2x + 1$ is a rational number.